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Non-Euclidean Mixture Model for Social Network Embedding

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Outline

- **Introduction and Preliminaries**
- Related Work: Social Network Embedding Models
- Methodology
- Experiment Results
- Ablation Studies
- Conclusions

Introduction and Preliminaries

- **Social networks** are omnipresent because they model interactions on social platforms
- Social network analysis is key to several applications (community detection, user connectivity)
- Widely agreed that social network links are formed from **homophily** or **social influence**
- **Homophily:** utilizes the intuition that associated nodes in a social network imply feature similarity, and an edge is usually generated between similar nodes (form cycles)
- **Social Influence:** the idea that popular nodes have direct influence in forming links e.g., users tend to follow celebrities (form hierarchies)

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Social Network Embedding Models

Observations:

- Shallow embedding models (structural embedding) do not effectively learn graph structure
- Models do not capture all social network factors e.g., social influence
- All network structures are modeled in the same space e.g., flat Euclidean space

Category	Description
Structural Embedding Models	**GraRep** [Cao *et al.*, 2015], shallow embedding integrating global structural information
	RolX [Henderson *et al.*, 2012], unsupervised learning approach using structural role based similarity
	GraphWave [Donnat *et al.*, 2018], shallow embedding model using spectral graph wavelet diffusion patterns
GNN Embedding Models (Euclidean space)	**GraphSAGE** [Hamilton *et al.*, 2017], inductive framework using node features and neighbor aggregation
	GCN [Kipf and Welling, 2017], semi-supervised learning model via graph convolution on local neighborhoods
	GAT [Veličković *et al.*, 2018], graph attention model using mask self-attention layers on local neighborhoods
Homophily-based Embedding Models	**GELTOR** [Hamedani *et al.*, 2023], embedding method using learning-to-rank with AdaSim* similarity metric
	NRP [Yang *et al.*, 2020], embedding model using pairwise personalized PageRank on the global graph
GNN Embedding Models (non-Euclidean space)	**HGCN** [Chami *et al.*, 2019], hyperbolic GCN model utilizing Riemannian geometry and hyperboloid model
	κ-GCN [Bachmann *et al.*, 2020], GCN model using product space e.g., product of constant curvature spaces
Mixture Models (homophily and social influence)	**RaRE** [Gu *et al.*, 2018], Bayesian probabilistic model for node proximity/popularity via posterior estimation
	NMM, our non-Euclidean mixture model (see Eqn. 9), without use of GraphVAE framework
	NMM-GNN , our non-Euclidean mixture model with non-Euclidean GraphVAE framework

SOTA baseline models

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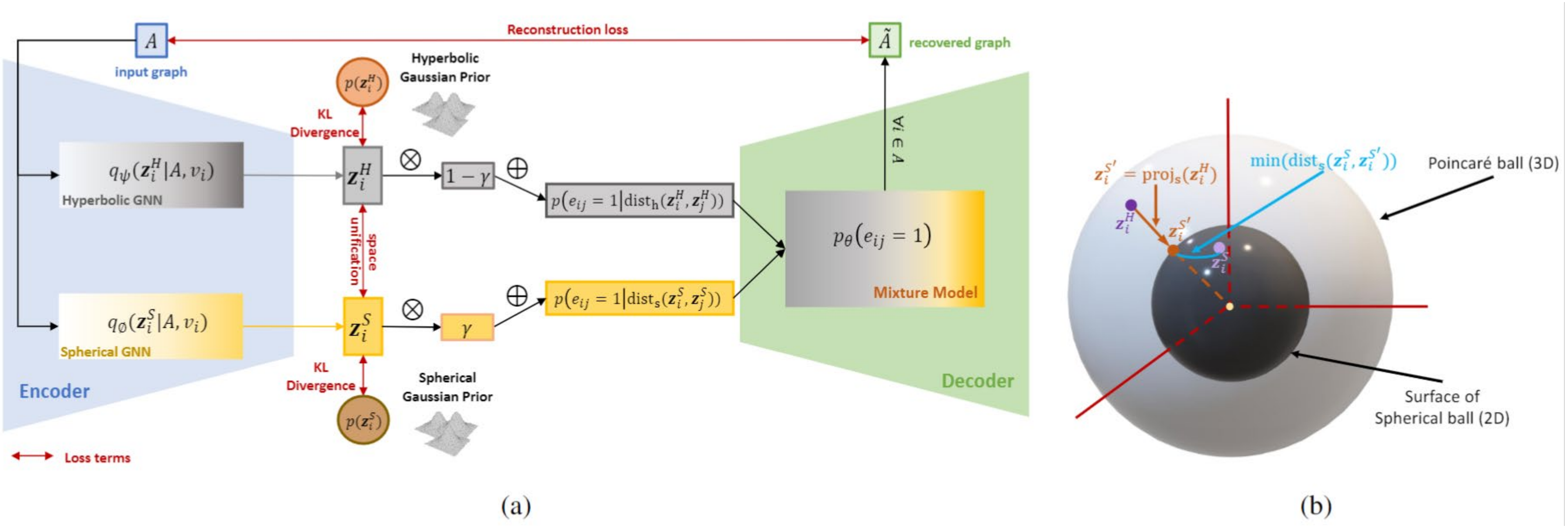
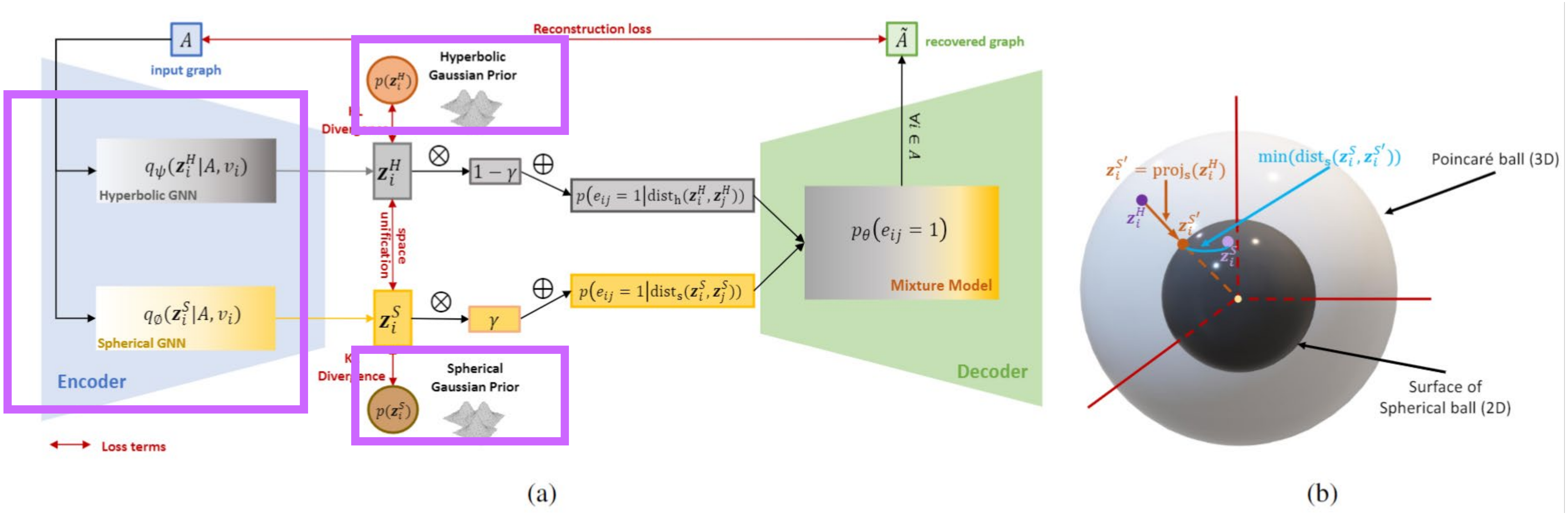
NMM-GNN Architecture

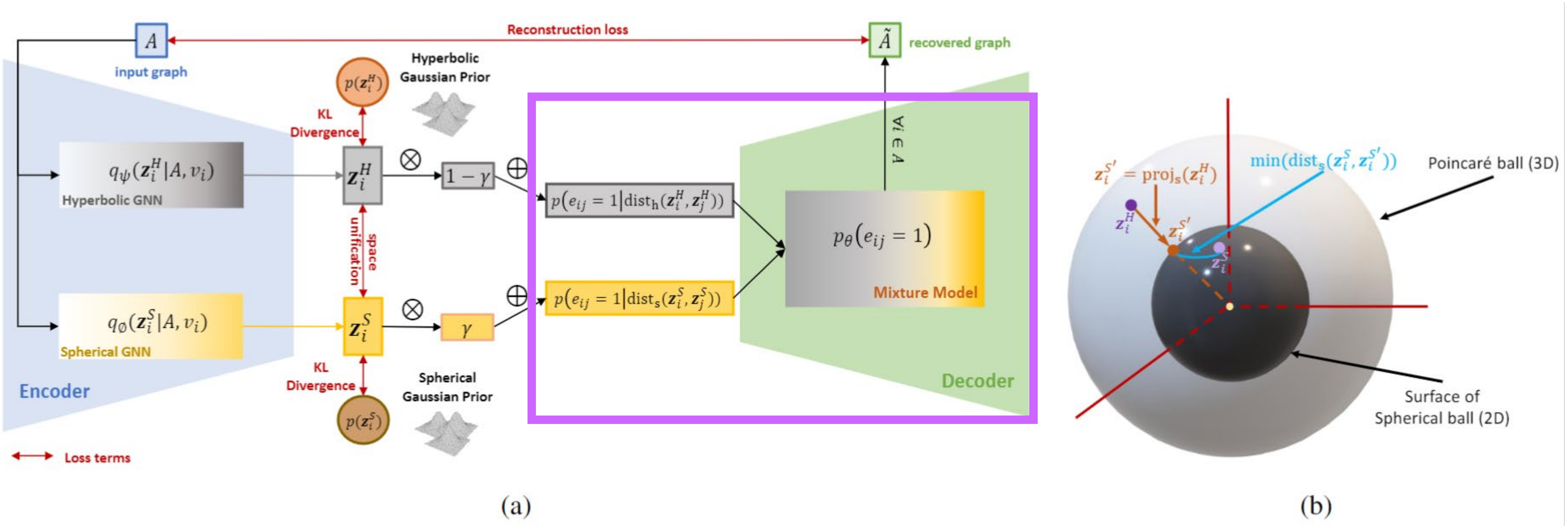


NMM-GNN Architecture



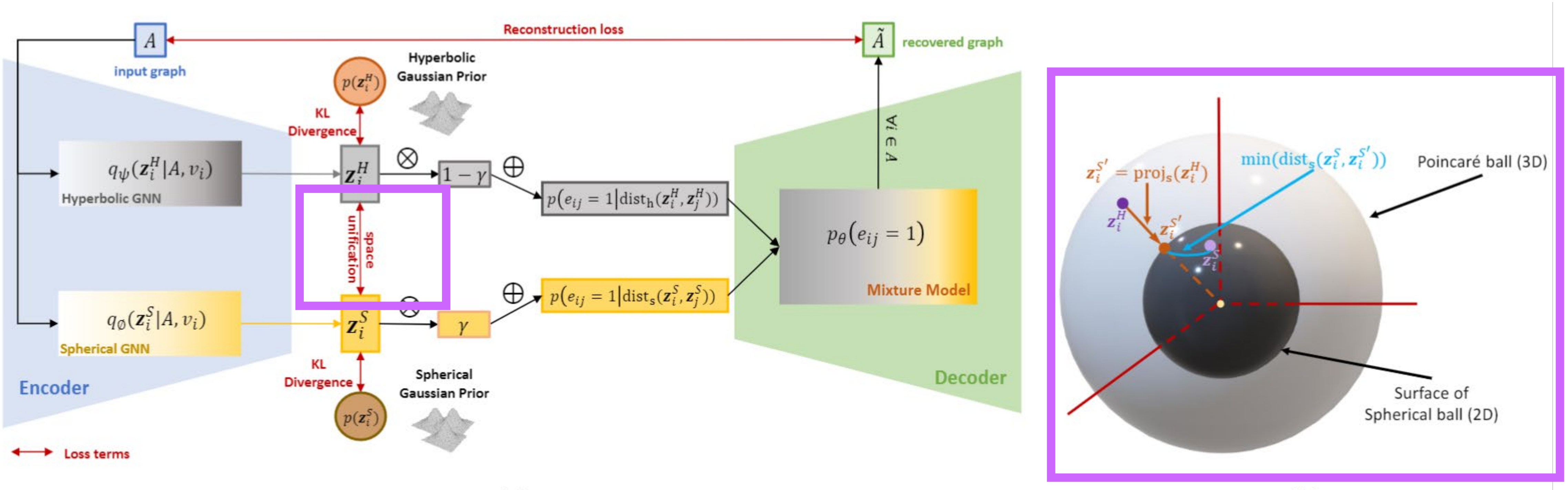
- The encoder maps nodes into z^S (homophily) and z^H (social influence), which follow non-Euclidean prior distributions.

NMM-GNN Architecture



- Embeddings are passed into our mixture model decoder (homophily + social influence).
- Objective: maximize likelihood to observe links, or equivalently minimize link reconstruction loss.

NMM-GNN Architecture



(a)

(b)

- We design a space unification component to align the distinct geometric spaces
  - Ensuring two embeddings of the same node are corresponding to each other

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Dataset Statistics

Table 1: Dataset statistics for evaluation datasets.

Dataset	# Vertices	# Edges	Type	# Classes
BlogCatalog	10.3K	334.0K	undirected	39
LiveJournal	4.8M	69.0M	directed	10
Friendster	65.6M	1.8B	undirected	–

Evaluation: Classification & Link Prediction

Table 3: Results of social network classification and link prediction for **Jaccard Index (%)**, **Hamming Loss (%)**, **F1 Score (%)**, **AUC (%)** using embedding dimension 64. Our **NMM** and its variants are in gray shading. For each group of models, the best results are bold-faced. The overall best results on each dataset are underscored. †Ablation study variant models using distinct non-Euclidean geometric spaces for **NMM** (homophily/social influence) where $\mathbb{E}$, $\mathbb{S}$, and $\mathbb{H}$ denote Euclidean, Spherical, and Hyperbolic spaces.

Datasets	BlogCatalog				LiveJournal				Friendster			
Metrics	Jaccard Index	Hamming Loss	F1 Score	AUC	Jaccard Index	Hamming Loss	F1 Score	AUC	Jaccard Index	Hamming Loss	F1 Score	AUC
GraRep	36.0	28.2	45.6	87.9	40.1	41.1	35.2	56.7	53.6	34.2	40.6	89.8
RolX	37.2	25.4	48.7	90.4	40.9	38.0	35.6	**60.1**	58.8	33.9	40.9	90.3
GraphWave	**39.5**	**22.8**	**48.9**	**92.3**	**42.2**	**37.6**	**35.9**	**60.1**	**59.0**	**31.5**	**41.1**	**90.5**
GraphSAGE	45.4	20.1	49.3	**92.0**	45.5	34.7	34.1	59.0	64.1	28.7	43.4	90.5
GCN	47.3	19.5	**55.1**	91.6	46.7	31.2	47.8	62.6	**66.5**	**28.0**	**47.2**	91.9
GAT	**47.9**	**19.3**	54.5	91.4	**47.4**	**28.5**	**49.0**	**65.3**	66.3	**28.0**	46.8	**92.0**
GELTOR	47.4	**19.3**	54.9	92.0	51.0	28.9	48.6	65.3	66.7	27.9	47.5	91.7
NRP	**61.6**	20.4	**65.2**	**95.5**	**69.7**	**24.5**	**64.0**	**78.7**	**72.2**	**22.6**	**52.8**	**92.2**
HGCN	56.7	**19.2**	60.9	92.7	58.8	**27.1**	**57.7**	68.5	**69.9**	24.3	49.9	**93.3**
κ-GCN	**61.6**	20.7	**65.4**	**95.3**	**63.6**	27.3	57.2	**69.1**	69.4	**24.1**	**50.3**	93.1
RaRE	61.4	20.6	65.6	95.1	74.2	23.8	65.1	79.9	75.7	22.5	55.0	94.4
NMM($\mathbb{H}^d/\mathbb{S}^d$)†	56.6	19.8	62.3	95.1	74.0	28.4	55.5	68.8	74.6	26.9	50.6	93.0
NMM($\mathbb{S}^d/\mathbb{S}^d$)†	57.1	19.6	65.9	94.0	74.7	27.6	57.1	69.0	75.3	26.2	52.5	93.4
NMM($\mathbb{E}^d/\mathbb{E}^d$)†	57.9	19.5	66.3	95.4	75.1	25.0	58.4	71.2	77.0	24.7	52.8	94.5
NMM($\mathbb{S}^d/\mathbb{E}^d$)†	59.2	19.2	67.1	95.5	75.3	24.4	59.3	74.5	77.5	23.3	54.3	94.5
NMM($\mathbb{H}^d/\mathbb{H}^d$)†	58.4	19.0	66.7	95.3	75.6	24.6	61.9	76.0	78.8	23.3	55.0	94.7
NMM($\mathbb{E}^d/\mathbb{H}^d$)†	60.3	19.1	67.8	95.7	76.2	23.2	64.4	79.2	79.1	22.6	55.4	94.5
NMM (ours)	**62.7**	19.0	70.9	95.8	76.5	22.7	**67.3**	84.2	79.8	22.1	56.3	94.8
NMM-GNN (ours)	62.6	**17.3**	**78.8**	**96.9**	**78.6**	**20.4**	**67.3**	**86.8**	**83.3**	**21.8**	**57.7**	**94.9**

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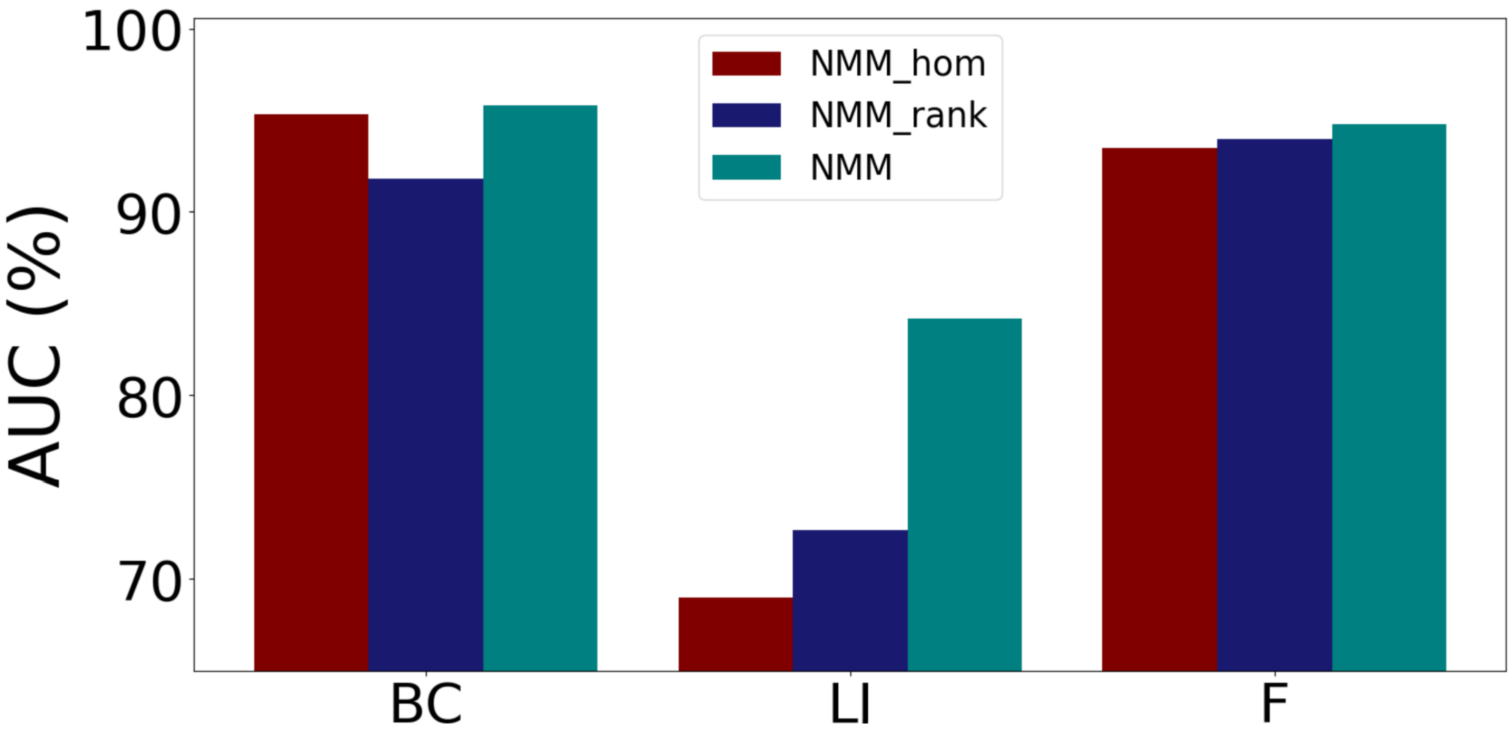
Ablation Study #1



- **Quality of using mixture model architecture: NMM_hom** (homophily deconstructed component), **NMM_rank** (social influence deconstructed component), and **NMM** (combined mixture model)

Ablation Study #2

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Datasets / Metrics	BlogCatalog Jaccard Index	BlogCatalog Hamming Loss	BlogCatalog F1 Score	BlogCatalog AUC	LiveJournal Jaccard Index	LiveJournal Hamming Loss	LiveJournal F1 Score	LiveJournal AUC	Friendster Jaccard Index	Friendster Hamming Loss	Friendster F1 Score	Friendster AUC
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- **Quality of using distinct non-Euclidean geometric spaces:** We study combinations of geometric spaces to model **NMM (homophily/social influence)** to observe the effect it has on learning topological structure, denoted with $\mathrm{NMM}(\cdot)^\dagger$

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NMM-GNN Contributions

(1) We propose Graph-based Non-Euclidean Mixture Model (NMM) to explain social network generation. NMM represents nodes via joint influence by homophily (spherical space) and social influence (hyperbolic space), while seamlessly unifying embeddings via space unification loss.

(2) To our knowledge, we are also the first to couple NMM with a graph-based VAE learning framework, NMM-GNN. Specifically, we introduce a novel non-Euclidean VAE framework where node embeddings are learned with a powerful encoder of GNNs using spherical and hyperbolic spaces, non-Euclidean Gaussian priors, and unified non-Euclidean optimization.

(3) Extensive experiments on several real-world datasets demonstrate effectiveness of NMM-GNN in social network generation and classification, which outperforms state-of-the-art network embedding models.

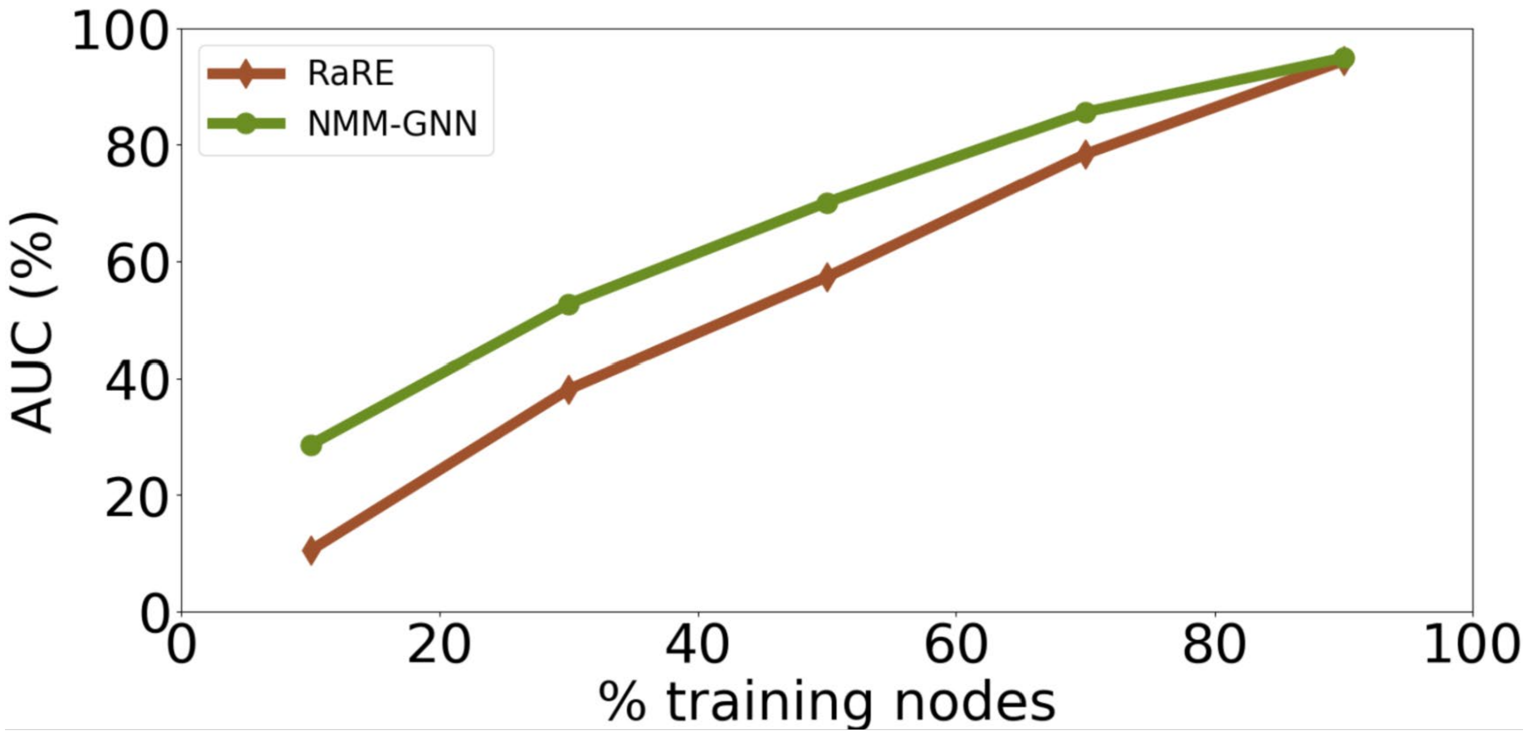
Extra

Motivation

(1) We aim to *understand* how the social network is generated e.g., which factors affect node connectivity and what topological patterns emerge in the network as a result.

(2) Using our learning from (1), we aim to design a more realistic deep learning model to *explain* how the network is generated (inferring new connections).

Ablation Study #3



- **Link prediction on unseen nodes (inductive task): NMM-GNN** outperforms **RaRE** on all settings of training nodes. Further, as less training nodes are observed, **NMM-GNN** outperforms **RaRE** by larger margins (e.g., 10\%~vs.~70\% training nodes), showing better generalization to unseen graphs.

Training: Parameter Optimization

- For homophily regulated nodes, parameter optimization is performed using Riemannian stochastic gradient descent (RSGD) for the spherical space.

- For social influence regulated nodes, parameter optimization is performed using RSGD for the hyperbolic space.

- Parameter optimization for γ in the decoder is performed using SGD.